FIG. 1

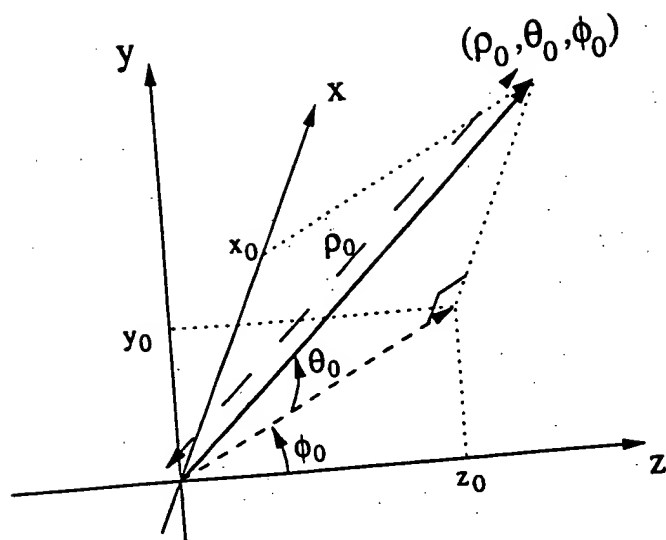


FIG. 2

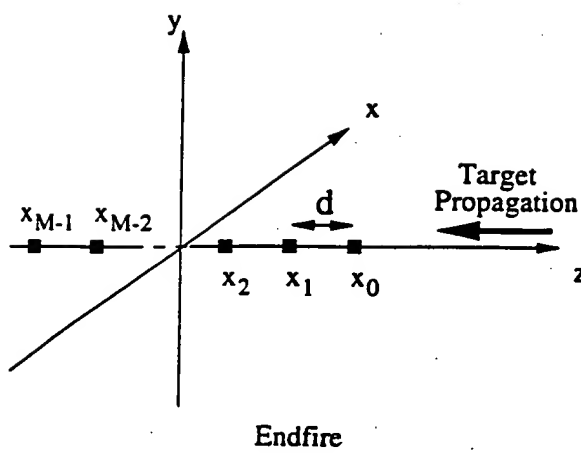


FIG. 3A

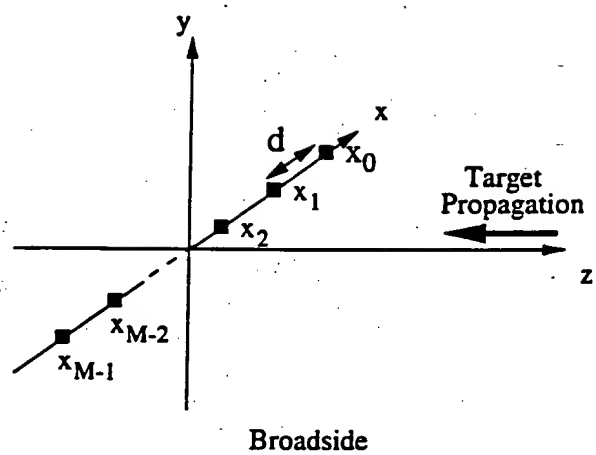


FIG. 3B

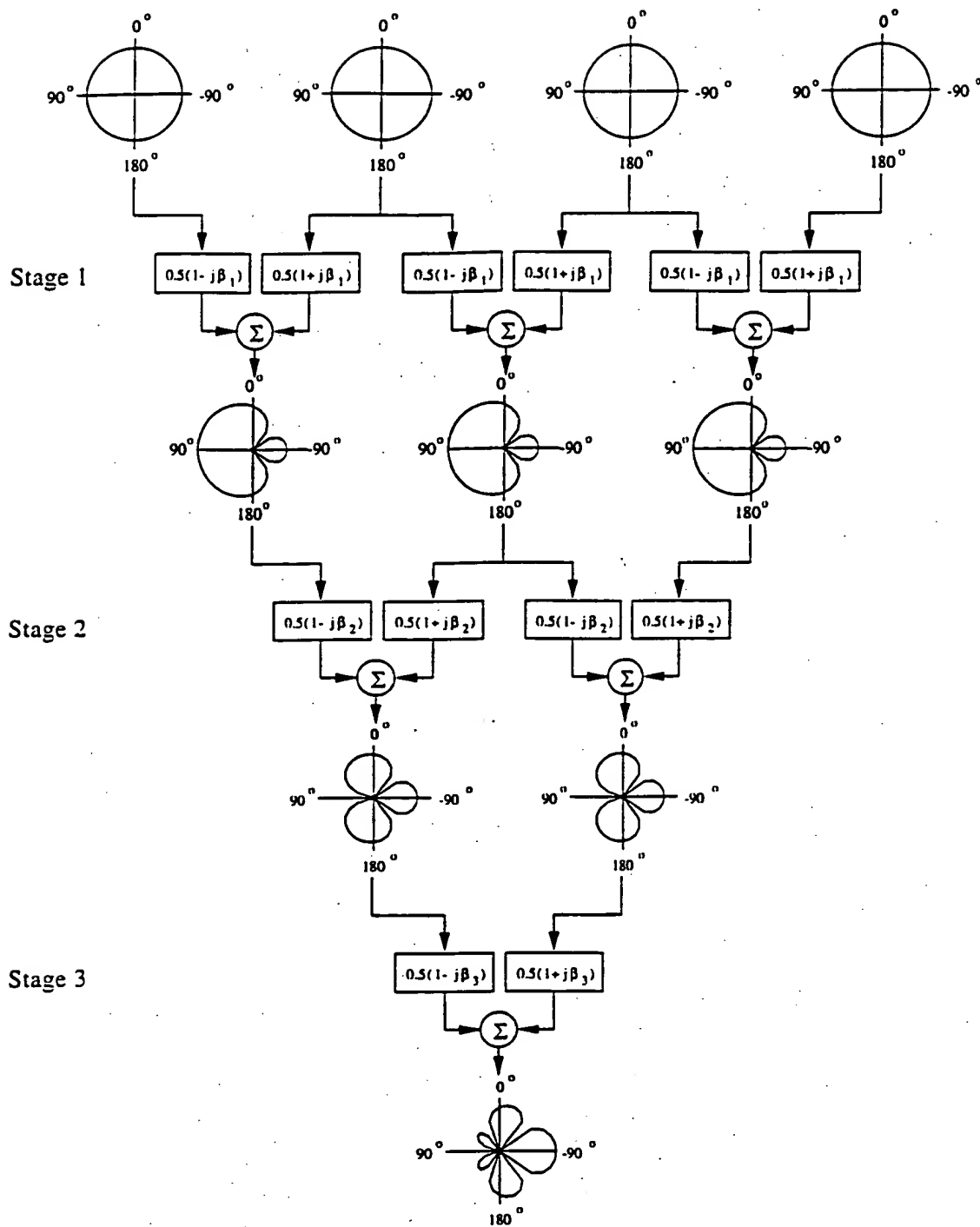


FIG. 4

664760"5479660

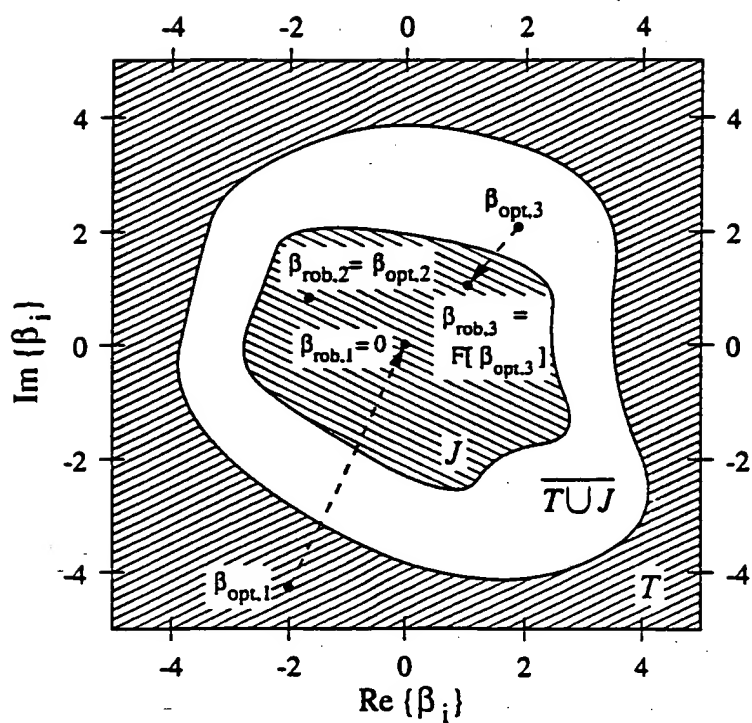


FIG. 5

664760" 5279660

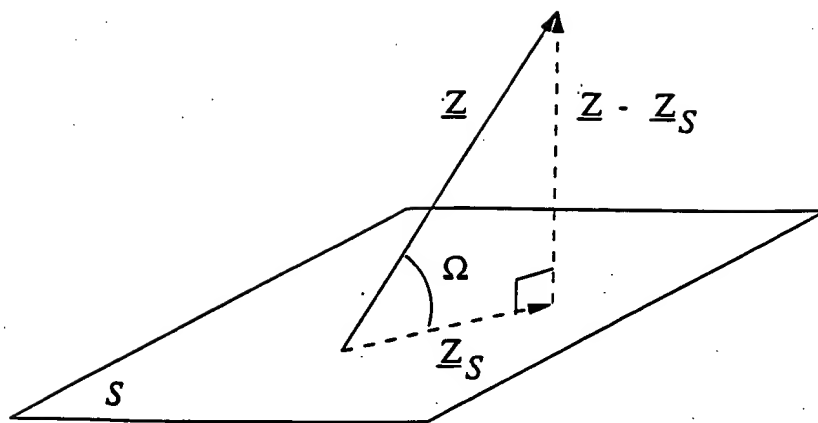
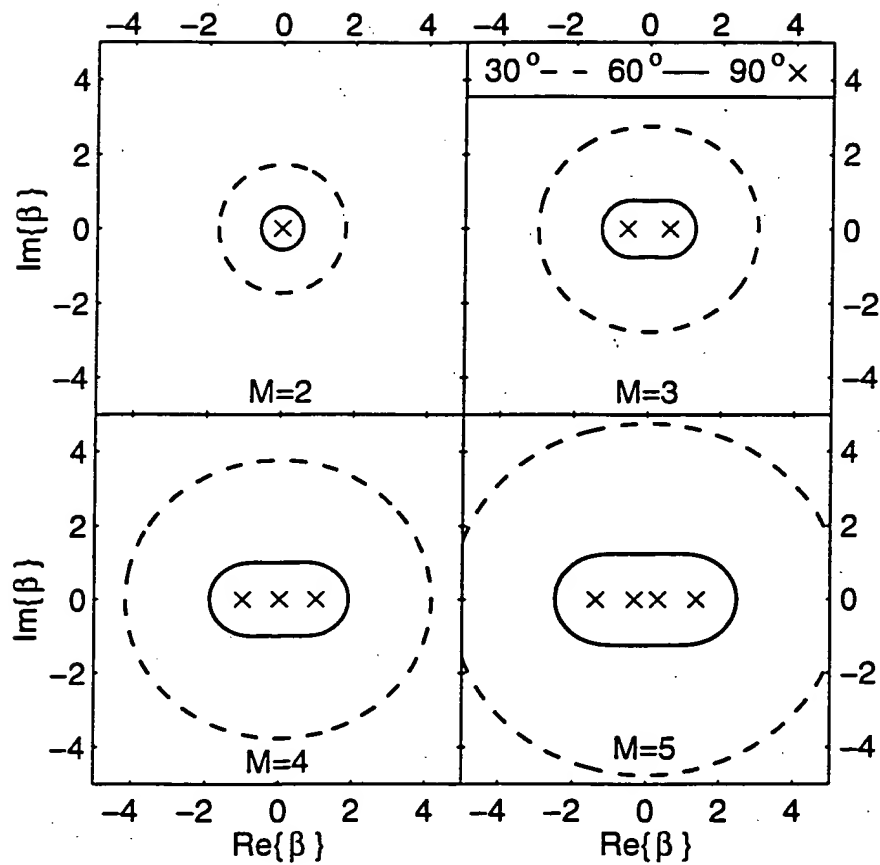


FIG. 6

66160° 5275660



Contours for $\Omega_i(\beta) = 30^\circ$ (---) 60° (—), and 90° (x) for $M = 2, 3, 4$, and 5 element arrays.

FIG. 7

664160" 52756260

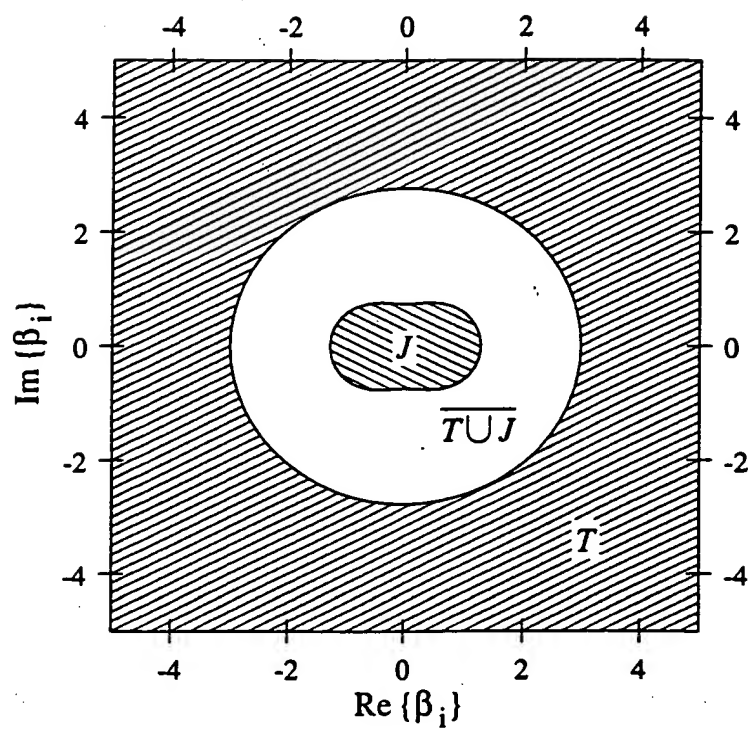
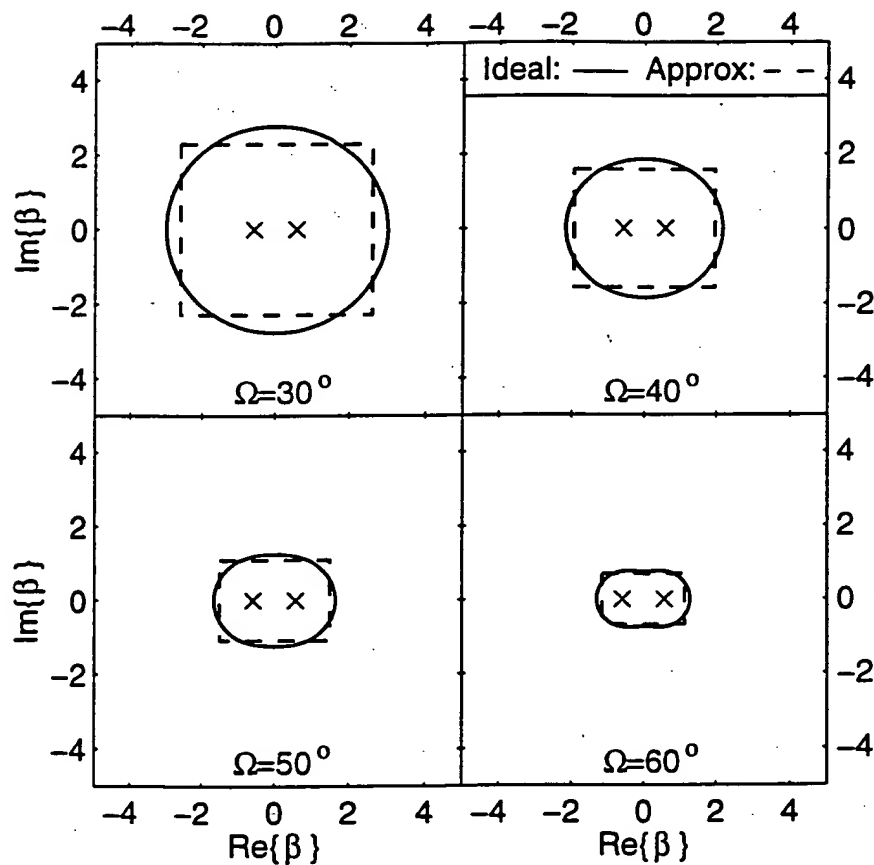


FIG. 8

661160 92796260



: Contours (—) and approximated contours (- - -) for $M=3$ and $\Omega(\beta_i) = 30^\circ, 40^\circ, 50^\circ$, and 60° , with β_i^\perp also indicated (x).

FIG. 9

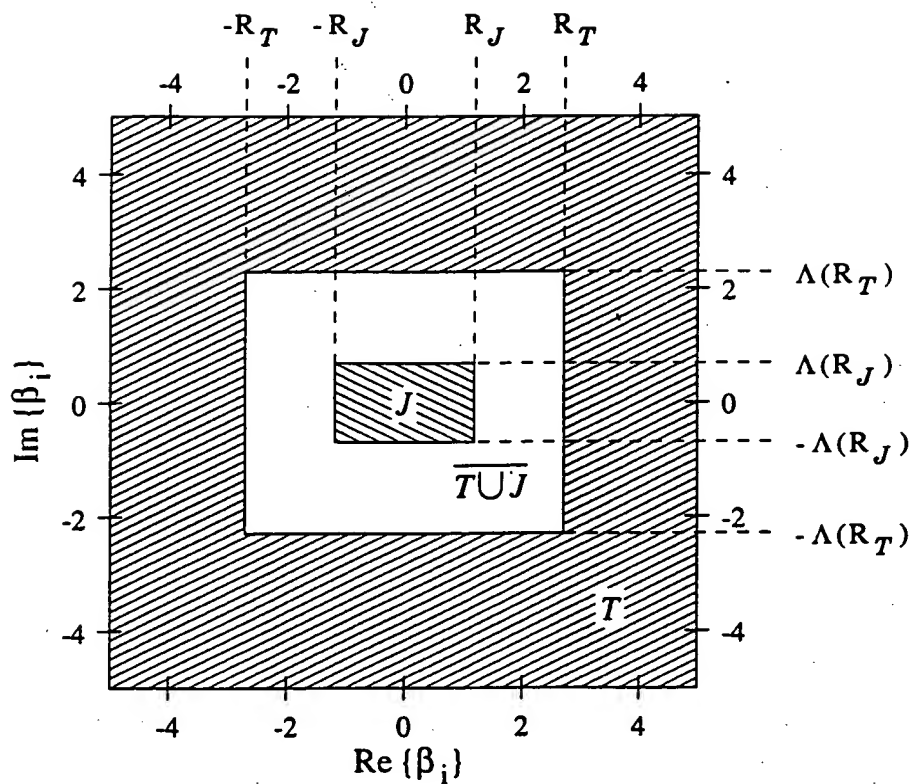
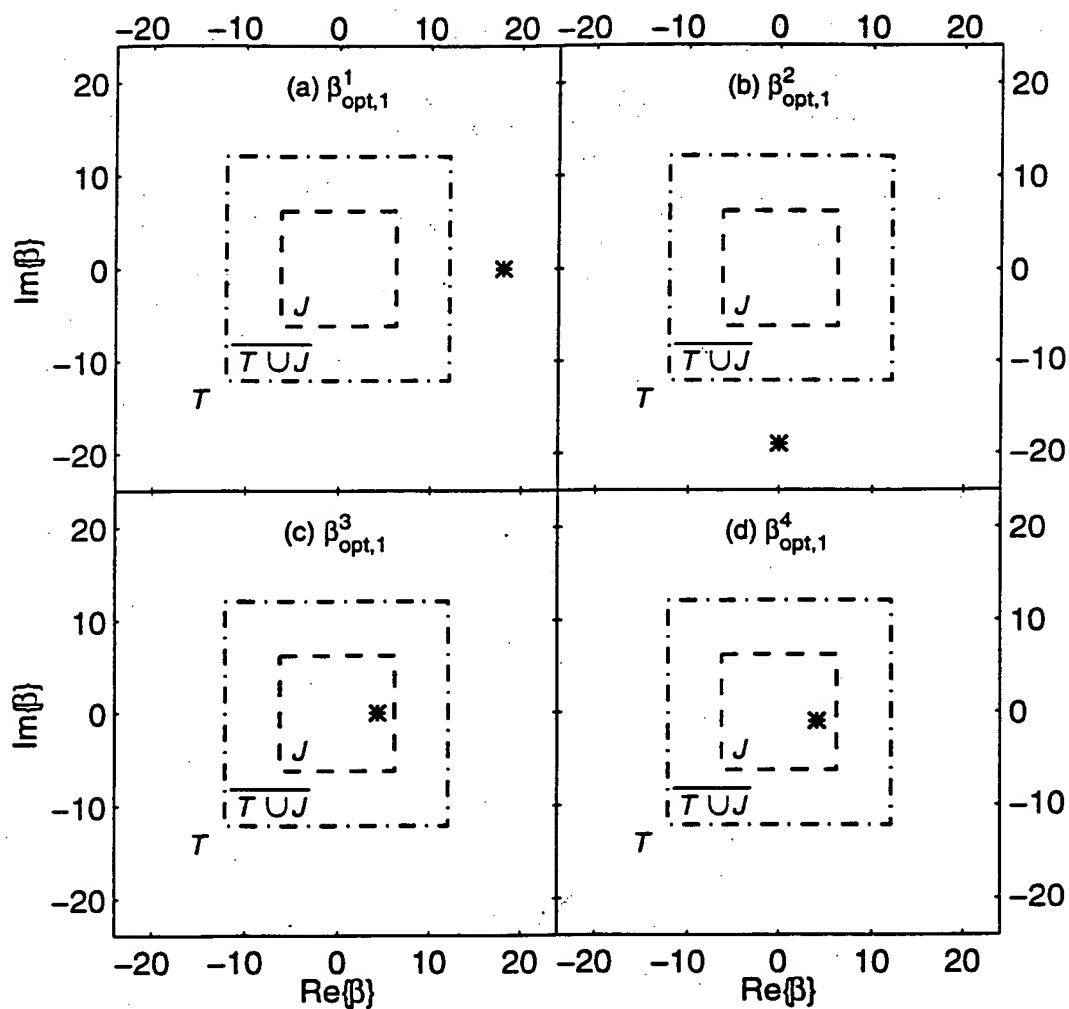


Diagram showing the three modified LENS parameter classification regions for an $M=3$ element array as according to Equation 3.23 when R_T and R_J are chosen to reflect the $\Omega_T = 30^\circ$ and $\Omega_J = 60^\circ$ contours, respectively.

FIG. 10



Four example values of $\beta_{\text{opt},1}$ for a two-element array along with the corresponding LENS robustness regions.

FIG. 11

$$\underline{X}$$

(Steered so that
 $\underline{H}_{\text{target}} = \underline{1}$)

Step 1: Solve for $\underline{\beta}_{\text{opt}}$ [Eq. 3.8]

$$\underline{\beta}_{\text{opt}} = \underset{\underline{\beta} \in \mathcal{C}}{\text{argmin}} \underline{W}_{\text{LENS},M}(\underline{\beta})^H \underline{R}_{XX} \underline{W}_{\text{LENS},M}(\underline{\beta})$$

Step 2: Make $\underline{\beta}_{\text{opt}}$ robust [Eqs. 3.30, 3.29]

$$\underline{\beta}_{\text{rob},i} = \bar{f}[\underline{\beta}_{\text{opt},i}] \underline{\beta}_{\text{opt},i}, \text{ where}$$

$$\bar{f}[\underline{\beta}_{\text{opt},i}] = \begin{cases} 0 & \underline{\beta}_{\text{opt},i} \in \mathcal{T} \\ 1 & \underline{\beta}_{\text{opt},i} \in \mathcal{J} \\ f[\underline{\beta}_{\text{opt},i}] & \underline{\beta}_{\text{opt},i} \in \overline{\mathcal{T} \cup \mathcal{J}} \end{cases}$$

$$f[\underline{\beta}_{\text{opt},i}] = \min \left[\frac{|\text{Re}\{\underline{\beta}_{\text{opt},i}\}| - R_{\mathcal{J}}}{R_{\mathcal{T}} - R_{\mathcal{J}}}, \frac{|\text{Im}\{\underline{\beta}_{\text{opt},i}\}| - \Lambda(R_{\mathcal{J}})}{\Lambda(R_{\mathcal{T}}) - \Lambda(R_{\mathcal{J}})} \right]$$

Step 3: Form weight vector [Eq. 3.1]

$$\underline{W} = \underline{W}_{\text{LENS},M}(\underline{\beta}_{\text{rob}})$$

\underline{W} for beamforming

FIG. 12

$$\underline{\beta}_{\zeta,opt}$$

Step 1: Transform to $\underline{\beta}_{ns,opt}$ [Eq. 5.4]

$$\beta_{ns,opt,i} = j \frac{(\beta_{\zeta,opt,i+j}) + \zeta(\beta_{\zeta,opt,i+j})}{(\beta_{\zeta,opt,i+j}) - \zeta(\beta_{\zeta,opt,i+j})}$$

Step 2: Standard LENS Robustness
Restriction [Eqs. 3.30, 3.29]

$$\beta_{ns,rob,i} = \bar{f}[\beta_{ns,opt,i}] \beta_{opt,i}, \text{ where}$$

$$\bar{f}[\beta_{ns,opt,i}] = \begin{cases} 0 & \beta_{ns,opt,i} \in \mathcal{T} \\ 1 & \beta_{opt,i} \in \mathcal{J} \\ f[\beta_{ns,opt,i}] & \beta_{ns,opt,i} \in \overline{\mathcal{T} \cup \mathcal{J}} \end{cases}$$

$$f[\beta_{ns,opt,i}] = \min \left[\frac{|\operatorname{Re}\{\beta_{ns,opt,i}\}| - R_{\mathcal{J}}}{R_{\mathcal{T}} - R_{\mathcal{J}}}, \frac{|\operatorname{Im}\{\beta_{ns,opt,i}\}| - \Lambda(R_{\mathcal{J}})}{\Lambda(R_{\mathcal{T}}) - \Lambda(R_{\mathcal{J}})} \right]$$

Step 3: Transform to $\underline{\beta}_{\zeta,rob}$ [Eq. 5.4]

$$\beta_{\zeta,rob,i} = j \frac{\zeta(\beta_{ns,rob,i+j}) + (\beta_{ns,rob,i+j})}{\zeta(\beta_{ns,rob,i+j}) - (\beta_{ns,rob,i+j})}$$

$$\underline{\beta}_{\zeta,rob}$$

FIG. 13

$$\underline{X} \quad \left(\begin{array}{l} \text{Steered so that} \\ H_{\text{target}} = 1 \end{array} \right)$$

Step 1: Skew Input [Eq. 5.1]

$$\underline{X}_\zeta = \underline{Z} \underline{X}$$

Step 2: Solve for $\underline{\beta}_{\zeta, \text{opt}}$ [Eq. 3.8]

$$\underline{\beta}_{\zeta, \text{opt}} = \arg \min_{\underline{\beta} \in \mathbb{C}^{M-1}} \underline{W}_{\text{LENS}, M}(\underline{\beta})^H R_{\underline{X}_\zeta \underline{X}_\zeta} \underline{W}_{\text{LENS}, M}(\underline{\beta})$$

Step 3: Make $\underline{\beta}_{\zeta, \text{opt}}$ robust

Step 4: Form weight vector [Eq. 3.1]

$$\underline{W} = \underline{W}_{\text{LENS}, M}(\underline{\beta}_{\zeta, \text{rob}})$$

\underline{W} for beamforming

FIG. 14

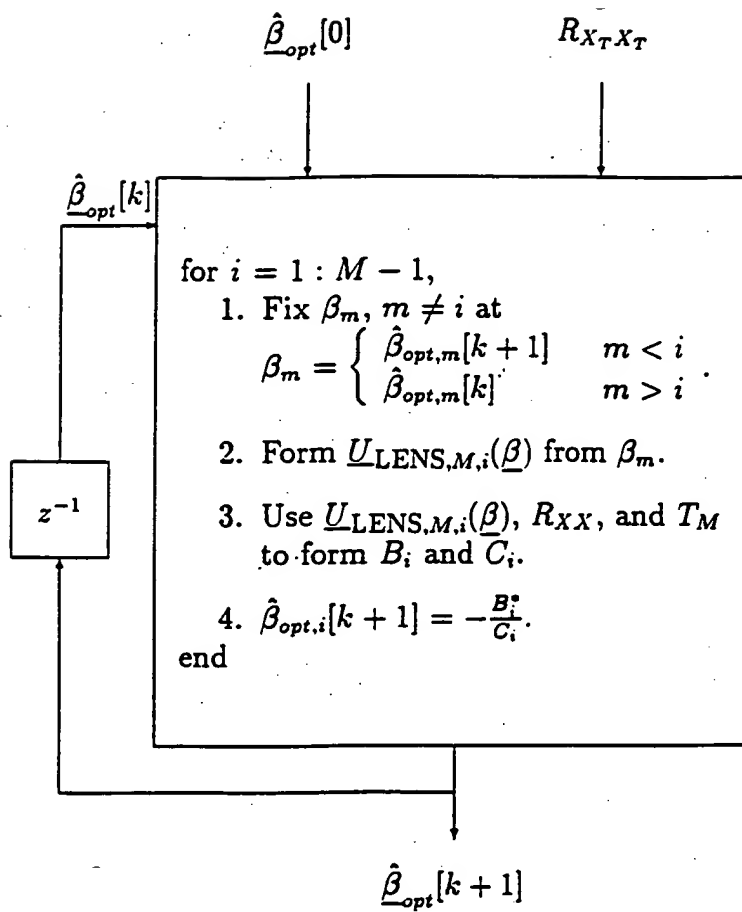
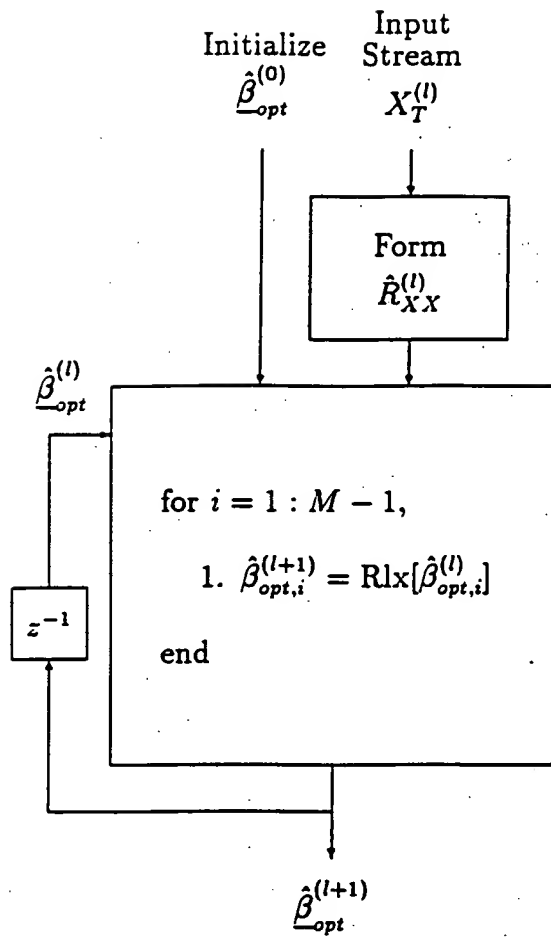
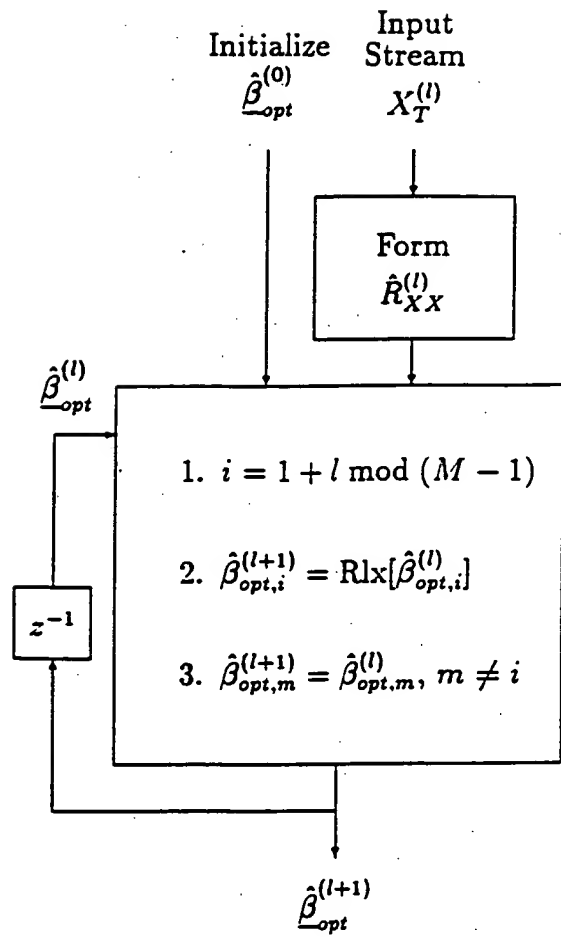


FIG. 15



(a) Complete Update



(b) Partial Update

FIG. 16

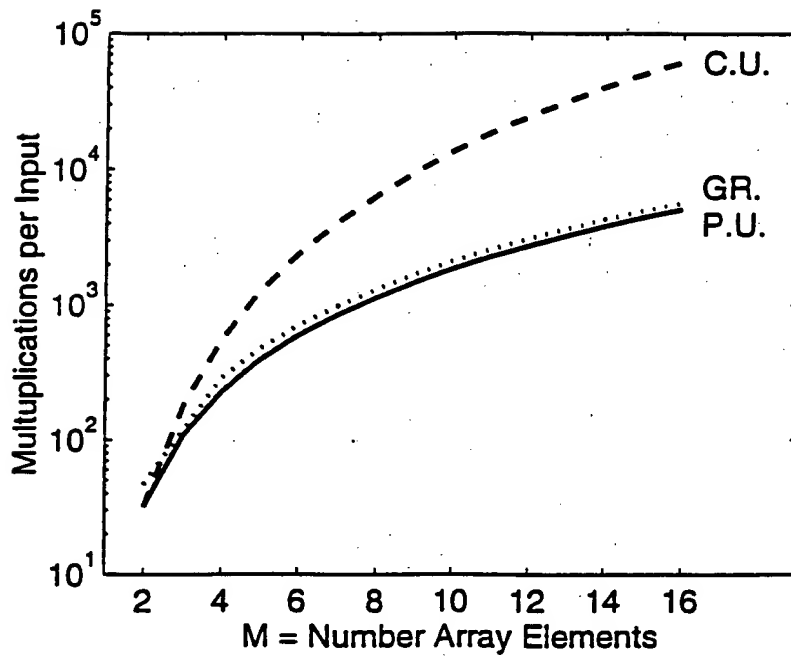


FIG. 17